New Modes on the Block: Security and Efficiency of Novel AO Compression Modes

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Hash functions and ZK-SNARKs

Cryptographic Hash Functions play a central role in SNARKs:

- In the Fiat-Shamir transform, enabling the 'N' in SNARK
- In the **Merkle Tree** commitment scheme:
  - Used in the ZK-STARK/FRI PCS.
  - In recursive SNARKs and IVC protocols.

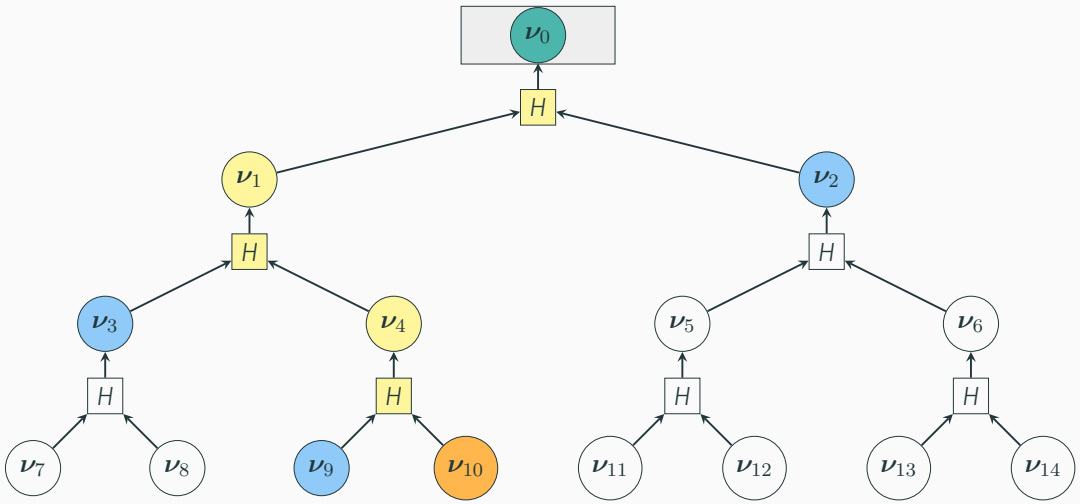
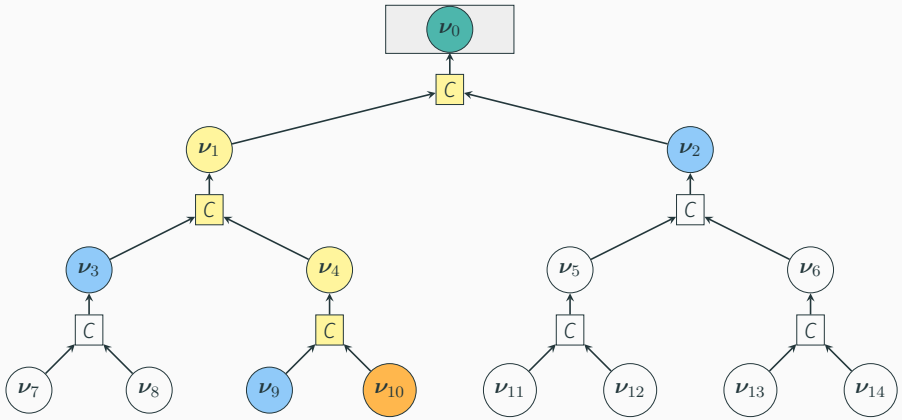


Figure 1: A binary Merkle Tree. Highlighted is an authentication path.

Merkle Trees Commitments

Merkle Trees used to instantiate a **vector commitment scheme**:

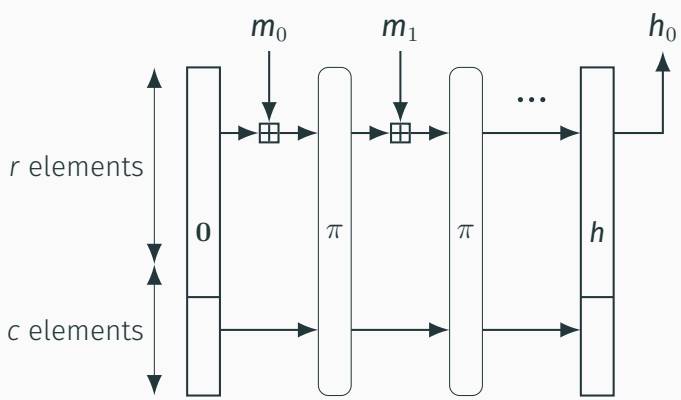
- The internal hash function is a $t:1$ compression function.
  - Usually $t = 2$.
- $\mathrm{Com}(\boldsymbol{m})$:
  - Split $\boldsymbol{m}$ into leaf nodes $\boldsymbol{m}_1 = \boldsymbol{\nu}_{2^h-1}, \ldots, \boldsymbol{m}_{2^h} = \boldsymbol{\nu}_{2^{h+1}-2}$.
  - Compute and output commitment $\boldsymbol{c} = \boldsymbol{\nu}_0$.
- $\mathrm{Open}(i)$: output $\boldsymbol{m}_i$'s co-path to the root.
  - Importantly: opening $\boldsymbol{\rho}$ is logarithmic in $|\boldsymbol{m}|$.
- $\mathrm{Ver}(\boldsymbol{c}, i, \boldsymbol{m}_i, \boldsymbol{\rho})$: compute $\boldsymbol{c}'$ from $(i, \boldsymbol{m}_i, \boldsymbol{\rho})$, accept iff $\boldsymbol{c}' = \boldsymbol{c}$.



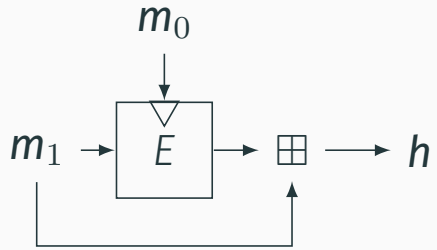
Compression Modes in the Wild

Which compression modes are used in practice?

- Blockcipher-based modes
  - PGV modes (e.g. Davies-Meyer, SHA-2).
- Permutation-based modes
  - Sponge (e.g. SHA-3).
    - Provably secure in the ideal-permutation model.
  - `Jive` (proposed with `Anemoi`).
  - Trunc (proposed with GRIFFIN, POSEIDON2, ...).
    - No provable security analysis.



Sponge



Davies-Meyer

Why Sponge?

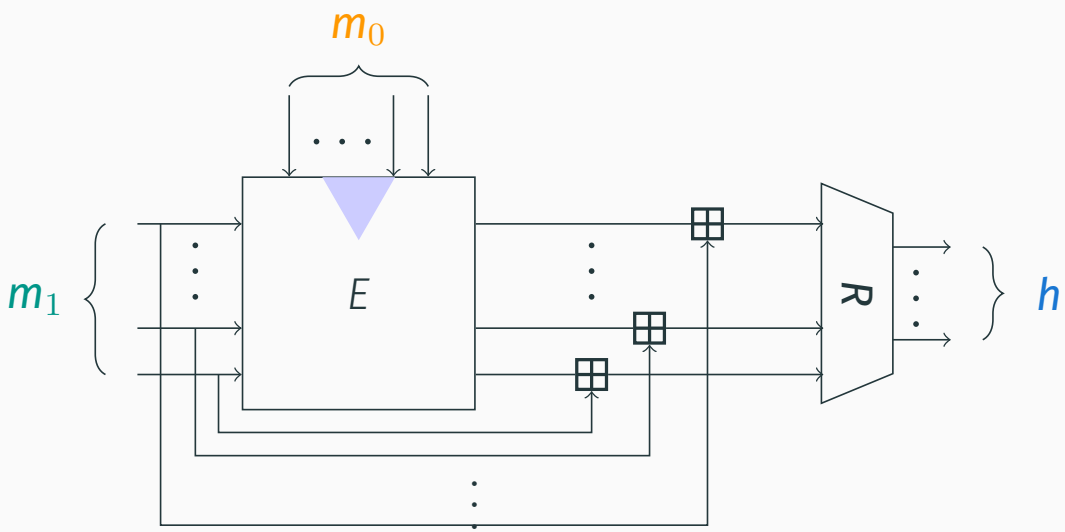
Sponge mode:

- RO-indifferentiable in the ideal permutation model.
- Single-round sponge (sponge-1):
  - CICO-1, CICO-k, CICO-(k_1, k_2)...
- Limitations of Sponge:
  - Suboptimal security bounds (especially collision/preimage resistance).
  - Most efficient?
- Many AO permutation (Poseidon-π, *Rescue*) are blockcipher-based.
  - Construct modes directly from the block cipher.
  - Better permutation-based modes.

The PGV-LC modes

At CSF'24, we introduced the PGV-LC **modes**:

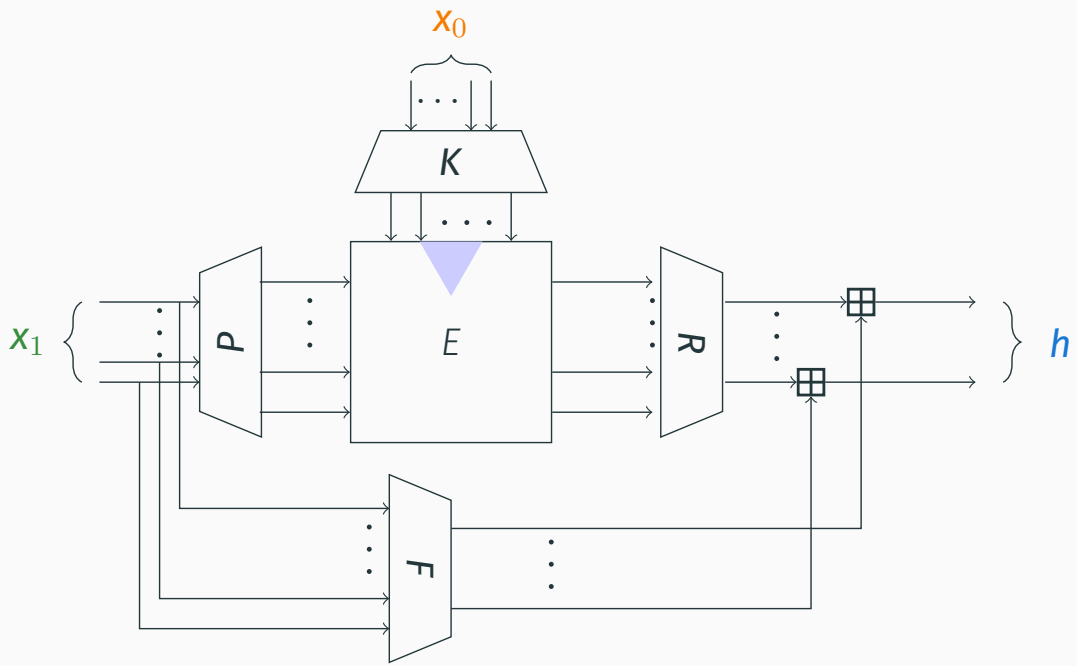
- Uses a blockcipher $E\colon \mathbb{F}_p^{\kappa} \times \mathbb{F}_p^n \to \mathbb{F}_p^n$.
- Matrix $\boldsymbol{R} \in \mathbb{F}_p^{\ell \times n}$ parametrizes output size.
  - ◇ Compresses its input $\Rightarrow \ell \leq n$.
  - ◇ Algebraic generalization of e.g. truncation and chopping.



The PGV-ELC mode

And an immediate extension thereof, called PGV-ELC:

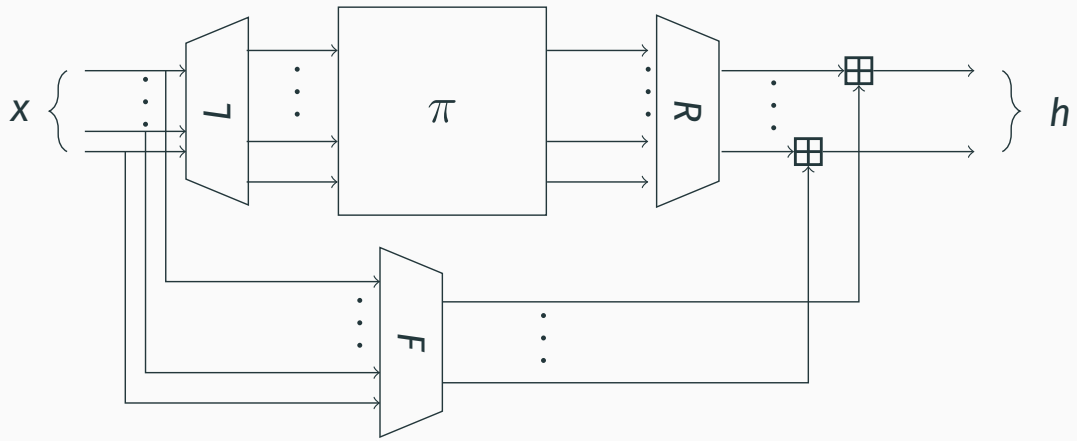
- Matrices $K \in \mathbb{F}_p^{\kappa \times \kappa'}$ and $P \in \mathbb{F}_p^{n \times n'}$ parametrize input size.
  - ◇ Expand their inputs $\Rightarrow \kappa' \leq \kappa$ and $n' \leq n$.
  - ◇ Algebraic generalization of e.g. zero-padding.
- Matrix $F \in \mathbb{F}_p^{\ell \times n'}$ adapts input to output size.



The PAX modes

We are also proposing the PAX **family of modes** [ongoing work]:

- Based on a permutation $\pi : \mathbb{F}_p^n \to \mathbb{F}_p^n$.
- $\mathcal{C}_{L,F,R,\pi}$ maps $\boldsymbol{x} \in \mathbb{F}_p^m$ to $\boldsymbol{h} \in \mathbb{F}_p^\ell$, with $\ell \leq m$.
- Expansion matrix $\boldsymbol{L} \in \mathbb{F}_p^{n \times m}$.
- Compression matrices $\boldsymbol{F} \in \mathbb{F}_p^{\ell \times m}$ and $\boldsymbol{R} \in \mathbb{F}_p^{\ell \times n}$.



PAX as a Generalization of Existing Modes

The PAX family generalizes most of the used constructions:

- $\mathrm{Trunc}\ C(x) = \lfloor \pi(x) + x \rfloor_\ell$:
  - Fix $n = m$, L, F, R are all (pseudo-)identities.
- `Jive` $C(x) = \sum_i x_i + \sum_i y_i$, with $y = \pi(x)$:
  - Fix $n = m$, L is the identity, F and R are circulant matrices on the repeated ith canonical vector of $\mathbb{F}_p^\ell$.
- First Sponge round: $C(x) = \lfloor \pi(x \parallel \mathbf{0}) \rfloor_\ell$
  - L, R pseudo-identities, $R = \mathbf{0}$.
  - Note: R is (clearly) not right-invertibile.

Provable Security

Black-box security of cryptographic constructions:

- Well-established model.
- Some primitives $\mathcal{P}_1, \ldots, \mathcal{P}_n$ used in a *mode*.
  - Often one primitive $\mathcal{P}$.
  - $\mathcal{P}$ is *ideal* (randomly sampled).
  - An adversary $\mathcal{A}$ can query the primitive(s).
- Security of a mode with respect to some property is expressed via an advantage function $\mathbf{Adv}(\mathcal{A}, q)$.

Compression Modes Security

For a compression mode $\mathcal{C}$:

- Often $\mathcal{P}$ is a blockcipher or a permutation.
  - Say over alphabet $\mathbb{F}_p$.
- Adversary makes q queries to $\mathcal{P}$.
- Some standard security properties:
  - Collision resistance:

$$\mathbf{Adv}^{\mathsf{COL}}_{\mathcal{C}}(\mathcal{A}, q) = \Pr\left[\mathcal{C}^{\mathcal{P}}(x) = \mathcal{C}^{\mathcal{P}}(x') \wedge x \neq x' \;\middle|\; (x, x') \leftarrow \mathcal{A}^{\mathcal{P}}()\right]$$

  - Preimage resistance:

$$\mathbf{Adv}^{\mathsf{PRE}}_{\mathcal{C}}(\mathcal{A}, q) = \Pr\left[\mathcal{C}^{\mathcal{P}}(x) = \mathcal{C}^{\mathcal{P}}(x') \;\middle|\; x \xleftarrow{\$} M; x' \leftarrow \mathcal{A}^{\mathcal{P}}\left(\mathcal{C}^{\mathcal{P}}(x)\right)\right]$$

  - Second-Preimage resistance (implied by collision resistance):

$$\mathbf{Adv}^{\mathsf{PRE2}}_{\mathcal{C}}(\mathcal{A}, q) = \Pr\left[\mathcal{C}^{\mathcal{P}}(x) = \mathcal{C}^{\mathcal{P}}(x') \wedge x \neq x' \;\middle|\; x \xleftarrow{\$} M; x' \leftarrow \mathcal{A}^{\mathcal{P}}(x)\right]$$

Random-Oracle Indifferentiability

Random-oracle indifferentiability is the ideal notion:

- Let $\mathcal{H}$ be a random oracle with compatible domain and range.
- Let $\mathcal{S}$ be a simulator algorithm
  - Makes $q_{\mathcal{S}}$ queries to $\mathcal{H}$, with $q_{\mathcal{S}}$ small.
- Let $\mathcal{D}$ be a differentiator:
  - Makes $q_{\mathcal{P}}$ queries to its left interface.
  - Makes $q_{\mathcal{H}}$ queries to its right interface.
- Then, letting $q = q_{\mathcal{P}} + q_{\mathcal{H}}$:

$$\mathbf{Adv}^{\mathsf{INDIF}}_{\mathcal{C},\mathcal{S}}(\mathcal{D}, q) = \left|\Pr\left[\mathcal{D}^{\mathcal{P},\mathcal{C}^{\mathcal{P}}}() = \top\right] - \Pr\left[\mathcal{D}^{\mathcal{S}^{\mathcal{H}},\mathcal{H}}() = \top\right]\right|$$

Merkle Tree Security

Which properties do we need for Merkle Tree commitments?

- Tree $\mathcal{T}$ using $C = \mathcal{C}^{\mathcal{P}}$ as the internal compression function.
  - $\mathcal{T}^C$ is also a compression function.
- Local opening security:

$$\mathbf{Adv}^{\mathrm{OPEN}}_{\mathcal{T}}(\mathcal{A}, q) = \Pr\left[\mathrm{Ver}\left(\mathcal{T}^C(m), i, m'_i, \rho'\right) = \top \wedge m'_i \neq m_i \;\middle|\; (m, i, m'_i, \rho') \leftarrow \mathcal{A}^{\mathcal{P}}()\right]$$

- It can be shown that:

$$\mathbf{Adv}^{\mathrm{OPEN}}_{\mathcal{T}}(q) \leq \mathbf{Adv}^{\mathrm{COL}}_{\mathcal{C}}(q)^1$$

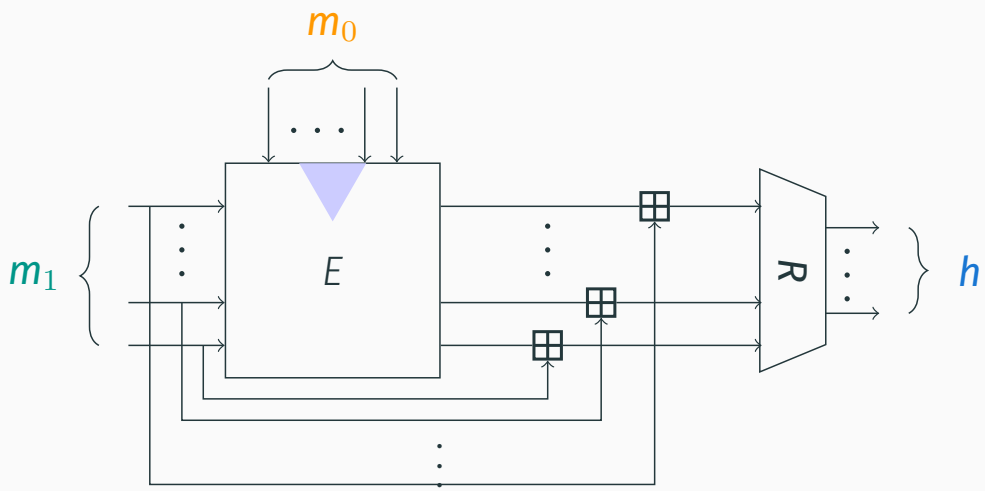
- Indifferentiability can however still be valuable.

[1] Or $2q$ in a slightly different notion.

PGV-LC Security

Collision resistance of PGV-LC:

1. Consider R right-invertible (full row rank).
2. $\mathbb{F}_p^n$ is partitioned into p^ℓ equivalence classes.
3. $\mathcal{A}$ can exploit partition unbalances from oracle replies.
4. Still, $\mathbf{Adv}_C^{\mathrm{COL}}(q) \leq \frac{q^2+q}{p^\ell - q}$ ($\approx$ birthday attack).
5. Similar reasoning for preimages: $\mathbf{Adv}_C^{\mathrm{PRE}}(q) \leq \frac{q}{p^\ell - q}$.



PGV-ELC Security

Collision resistance of PGV-ELC:

1. Consider K and P left-invertible, F right-invertible.
   - Also induce partitions of respective spaces.
2. 'Meaningless' queries, can be exploited only indirectly.
3. Again, $\mathbf{Adv}_C^{\mathrm{COL}}(q) \leq \frac{q^2+q}{p^\ell - q}$.
4. And for preimage resistance: $\mathbf{Adv}_C^{\mathrm{PRE}}(q) \leq \frac{q}{p^\ell - q}$.

Security of PAX

Indifferentiability of PAX based on the following simulator:

function $\mathcal{S}^{\mathcal{H}}(b \in \{-1,1\}, z \in \mathbb{F}_p^n) \to \mathbb{F}_p^n$

- **static** $T \leftarrow \emptyset$
- **if** $b = 1$ **then** ▷ *Forward query*
  - **if** $z \in \mathrm{Dom}(T)$ **then return** $T(z)$
  - $v \leftarrow L^{+}z$
  - **if** $z = Lv$ **then** ▷ *Queried point has a preimage*
    - $h \leftarrow \mathcal{H}(v)$
    - $y \xleftarrow{\$} [h - Fv]_R \setminus \mathrm{Ran}(T)$ ▷ $[x]_R$ *equiv. class of* x
  - **else** ▷ *Queried point is preimage-free*
    - $y \xleftarrow{\$} \mathbb{F}_p^n \setminus \mathrm{Ran}(T)$
  - $T \leftarrow T \cup \{(z, y)\}$
  - **return** y
- **else** ▷ *Backward query*
  - **if** $z \in \mathrm{Ran}(T)$ **then return** $T^{-1}(z)$
  - $x \xleftarrow{\$} \mathbb{F}_p^n \setminus \mathrm{Dom}(T)$
  - $T \leftarrow T \cup \{(x, z)\}$
  - **return** x

Modes security comparison

Adversarial advantages for a compression function $\mathbb{F}_p^m \to \mathbb{F}_p^\ell$:

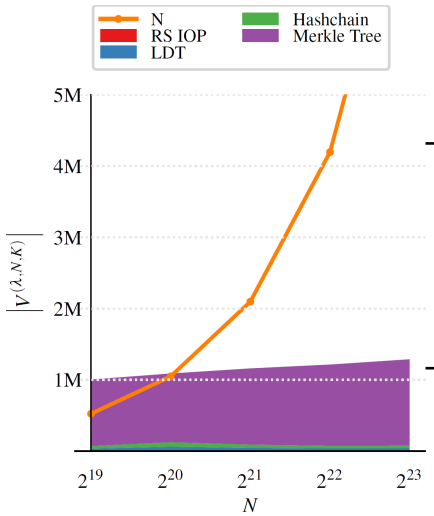
Mode	Primitive	COL	PRE	DIF
PGV-ELC	Block$(m/2+c, m/2+c)$	q^2/p^ℓ	q/p^ℓ	q/p^c
PAX	Perm$(m+c)$	q^2/p^ℓ	q/p^ℓ	q/p^c
Sponge-1	Perm$(m+c)$	$\min\{q^2/p^\ell, q/p^c\}$	$q/p^{\min\{\ell,c\}}$	q/p^c
Sponge	Perm$(r+c)$	$q^2/p^{\min\{\ell,c\}}$	$q/p^{\min\{\ell,c\}}$	q^2/p^c

- PAX and PGV-LC have optimal COL and PRE resistance.
- Single-iteration sponge (Sponge-1) has better bounds than multi-iteration.
- COL and PRE resistance of Sponge are sub-optimal.
- PGV-ELC indifferentiability does not depend on the key 'capacity'.

Impact of Hashing

How much is hashing relevant in a SNARK computation?

- In FRACTAL's verifier circuit [6], about 90% of constraints are from hash functions.
- In the RISC Zero chess example [13], about 37% of total time is spent on hashing.
- *Circuit* and *plain* performance are both important.
  - ZKProofMarket: by 2030 2^{27} proofs per day.
  - Even a 1ms saving per proof: $\approx 2^{24}$ seconds per day.



N	2^{20}	2^{21}	2^{22}	2^{23}
RS-IOP	2,948	3,036	3,124	3,212
LDT	58,586	42,658	36,190	37,598
Hashchain	65,184	46,878	37,566	37,566
Merkle Tree	962,544	1,068,122	1,137,818	1,210,418
- leaf hash	235,840	235,642	284,746	309,298
- 2-to-1 hash	663,830	760,624	781,216	829,264
- cap hash	62,874	71,856	71,856	71,856
Total	**1,132,666**	**1,210,292**	**1,264,299**	**1,338,406**

Caveat: Heuristic Security

Instantiating a compression functions:

- Target a specific security level (e.g. 128-bits for collision resistance).
- Choose a concrete underlying primitive:
  - parametrization based on cryptanalytical bounds.
- For PGV-ELC modes, this is not completely clear.
  - Need for cryptanalysis on AO blockcipher-based compression.
  - Experiments still relevant to establish efficiency margin.
- For PAX modes, results about previous permutation-based modes apply.

Arithmetization-Oriented Hash Functions

How to address the bottleneck?

- Plain performance: traditional solutions (e.g. SHA) work well.
- Circuit performance:
  - Arithmetic circuit over a field $\mathbb{F}_{p^d}$.
  - In SNARKs that rely on pairing-friendly EC: $p \approx 2^{256}$, $d = 1$.
  - For STARKs that rely on FRI IOPP: $p \approx 2^{32}$, $p \approx 2^{64}$, $d = 1$.
  - Multiplicative complexity/depth is an important starting point.
  - But cost is really tied to the *arithmetization technique*.
- *Arithmetization-Oriented* hash functions.

Arithmetization Techniques

Which arithmetization technique?

- R1CS arithmetization:

$$Ax \odot Bx = Cx$$

- (basic) $\mathcal{PlonK}$ arithmetization:

$$\left\{ q_{\mathrm{x}} x + q_{\mathrm{y}} y + q_{\mathrm{z}} z + q_{\mathrm{xy}} xy + q_c = 0 \right.$$

- AIR arithmetization:

$$T(x_{\mathrm{in}}, x_{\mathrm{out}}) \qquad \left\{ B(x) \right.$$

- Lookup arguments: efficient range-checks and more.
- Traditional solutions become arithmetization-oriented?

PGV-LC **Proof Generation**

Time to generate a MT opening proof on Groth16:

- Scalar field of the BLS12-381 elliptic curve: $\log_2(p) \approx 255$.

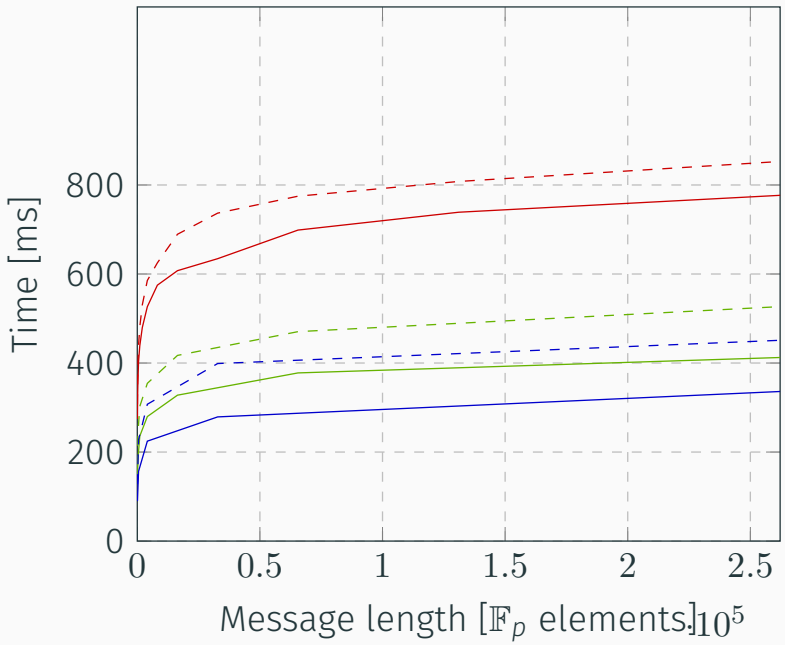


Figure 3: Dashed: Poseidon, Solid: Hades with PGV-LC

PGV-LC **Root Computation**

Time to compute the root of a Merkle Tree:

- Scalar field of the BLS12-381 elliptic curve: $\log_2(p) \approx 255$.

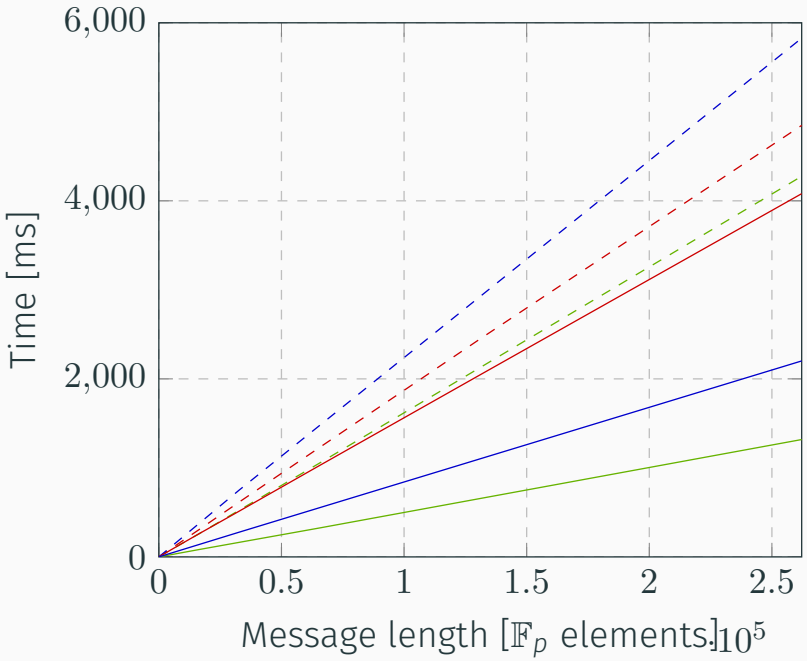


Figure 4: Dashed: Poseidon, Solid: Hades with PGV-LC

Plain performance

Comp. ratio	BN254 PAX	BN254 Sponge	Goldilocks PAX	Goldilocks Sponge
		Poseidon-π		
$2:1$	$9.4\,\mu s$	$17.5\,\mu s$	$3.5\,\mu s$	$7.3\,\mu s$
$4:1$	$28.2\,\mu s$	$42.4\,\mu s$	$12.2\,\mu s$	$18.6\,\mu s$
$8:1$	$103.7\,\mu s$	$130.4\,\mu s$	$141.5\,\mu s$	$265.4\,\mu s$
		Rescue		
$2:1$	$334.6\,\mu s$	$353.2\,\mu s$	$16.8\,\mu s$	$26.2\,\mu s$
$4:1$	$372.0\,\mu s$	$382.9\,\mu s$	$36.2\,\mu s$	$46.9\,\mu s$
$8:1$	$553.5\,\mu s$	$626.1\,\mu s$	$117.7\,\mu s$	$153.5\,\mu s$

Groth16 benchmarks

Preimage circuit in Groth16:

Comp. ratio	# R1CS constraints PAX	# R1CS constraints Sponge	Proof Generation time PAX	Proof Generation time Sponge
		Poseidon-π		
$2:1$	**221**	246	44.5 ms	47.3 ms
$4:1$	**268**	293	51.3 ms	54.9 ms
$8:1$	**368**	393	66.3 ms	71.5 ms
		Rescue		
$2:1$	**240**	252	41.2 ms	42.3 ms
$4:1$	**264**	270	44.9 ms	45.1 ms
$8:1$	**384**	432	63.8 ms	67.7 ms

Plonky2 benchmarks

Preimage circuit in *Plonky2*:

Comp. ratio	# gates		Proof Generation time	
	PAX	Sponge	PAX	Sponge
Poseidon-π				
$2:1$	**122**	259	11.3 ms	16.5 ms
$4:1$	**439**	668	26.0 ms	27.1 ms
$8:1$	**2065**	2864	90.8 ms	92.9 ms
Rescue				
$2:1$	**91**	175	10.9 ms	17.2 ms
$4:1$	**284**	418	16.8 ms	27.1 ms
$8:1$	**976**	1213	47.9 ms	49.0 ms

Roll credits

Fin

Thank you for your attention!

Any questions?

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